Letters

Comments on "A Unique Extraction of Metamaterial Parameters Based on Kramers–Kronig Relationship"

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The above paper [1] presents a numerical procedure for extracting the effective constitutive parameters of metamaterials. From the fact that the imaginary part of the refractive index is explicitly given by the magnitude of the transmission coefficient, while the real part is ambiguously defined by the phase of the transmission coefficient, the method above enforces causality to calculate the real part of the refractive index from its imaginary part by numerically integrating the Kramers–Kronig relation over a prescribed frequency range. The refractive index thus calculated for a metamaterial composed of split-ring resonator (SRR)-wire unit cells shows some discontinuities appearing on the covered frequency interval.

The authors [1] interpret the discontinuity in the refractive index as an upper limit of the effective medium theory. For example, in [1, Fig. 9(b)], the gray areas are claimed to represent frequency regions above this upper limit. It is argued that the rapidly changing phase of the transmission coefficient greatly affects the real part of the refractive index for the fundamental branch, prohibiting the retrieval of effective material parameters at low frequencies. They also conclude that the discontinuity of the refractive index indicates that the limit of the effective medium theory has been reached, thus placing limitations on transmission-reflection-based methods to retrieve constitutive parameters of metamaterials containing several layers of a unit cell.

The purpose of this letter is to point out that the cause of the discontinuities is due to improperly calculated branch indices, denoted as m in [1]. To show this, we recast the complex transmission coefficient expression $T = |T| \exp(i\phi) = \exp(\mathrm{i}nk_0\, d)$ to obtain for the refractive index

$$n = \frac{1}{k_0 d}(-i \ln |T| + \phi), \qquad -\pi < \phi < \pi \tag{1a}$$

$$n = \frac{1}{k_0 d}(-i \ln T + 2m\pi), \qquad \pi \le \phi \le -\pi \tag{1b}$$

where k_0 is the free-space wavenumber, d is the sample length, and the integer m denotes the branch index. The phase $2m\pi$ is added to the complex T to make the logarithmic function single valued and continuous at $|T|$.

To verify the results shown in [1, Fig. 9], we calculate the transmission coefficient T for a 17.5-mm-thick metamaterial sample with assigned electric permittivity and magnetic permeability represented by the Drude and Lorentz models as $\varepsilon = 4.5[1 - f_p^2/(f^2 + \gamma_e f)]$, $\mu = 1 - F_m f^2/(f^2 - f_m^2 + i\gamma_m f)$ with $f_p = 12.5$ GHz, $\gamma_e = 0.5$ GHz, $F_m = 0.3$, $f_m = 9.5$ GHz, and $\gamma_m = 0.2$ GHz such that $\mathrm{Im}\{\varepsilon\} > 0$ and $\mathrm{Im}\{\mu\} > 0$ at all frequencies as required by the passivity condition, consistent with the assumed time harmonic variation $\exp(-i\omega t)$. The quantities above have been selected and carefully adjusted to well reproduce the scattering parameters obtained from electromagnetic simulation for the unit cell described in [3], and which is the same as that considered in [1]. The unit cell is in the shape of a cube, which is repeated periodically to build in free space a cubic metamaterial of lattice spacing (= 2.5 mm) equal to the cube edge length. The thickness of 17.5 mm (seven-unit cells) is that of the slab whose retrieved refractive index is shown in [1, Fig. 9].

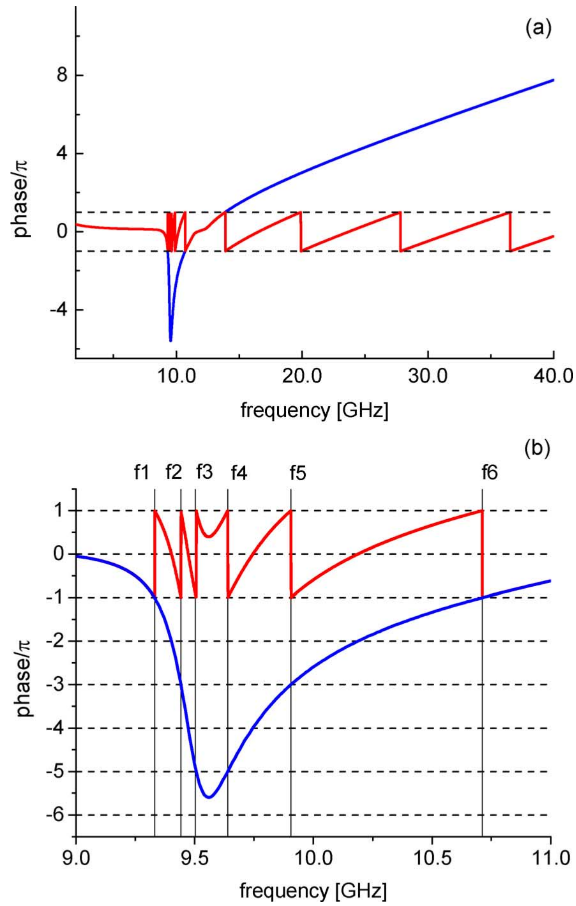

Fig. 1. Wrapped (red in online version) and unwrapped (blue in online version) phases of the transmission coefficient over the: (a) 2–40-GHz range and (b) zoomed in on the 9–11-GHz range.

From the calculated transmission coefficient phase, which is displayed in Fig. 1, we identify over the 2–40-GHz frequency range ten resonance frequencies at which the phase of T jumps from $-\pi$ to π.

In unwrapping the phase of the transmission coefficient [2], the frequency interval $f1$–$f2$ (Fig. 1) is shifted down by -2π, the second interval $f2$–$f3$ by -4π, the third one $f3$–$f4$ by -6π, the next two intervals ($f4$–$f5$ and $f5$–$f6$) are shifted down by -4π and -2π, respectively; the sixth interval $f6$–$f7$ is zero shifted, while the next three intervals are consecutively shifted up by $+2\pi$. Therefore, the branch indices m are calculated (Fig. 2) from the operation to make the phase of T continuous. Once the complex transmission coefficient T has

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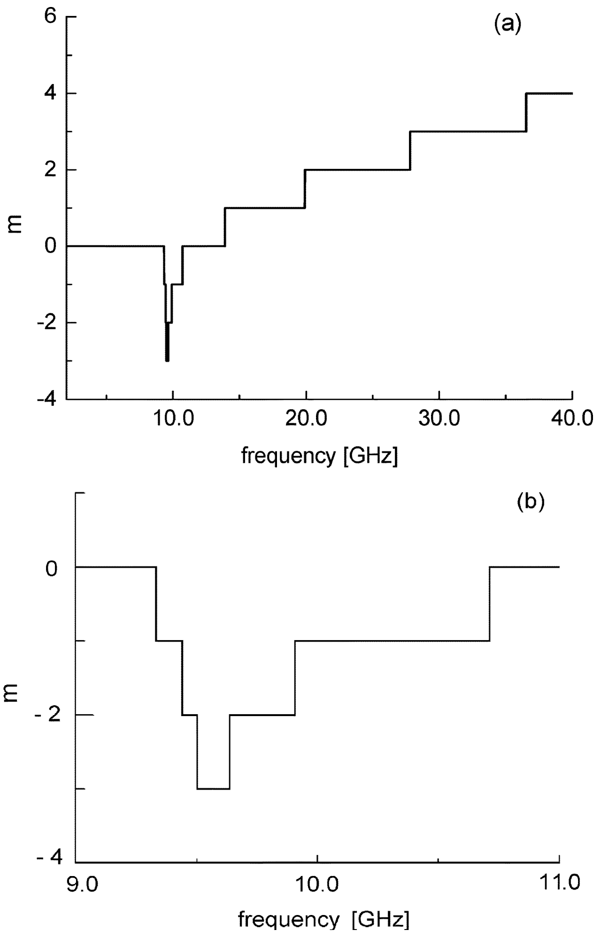

Fig. 2. For the 1.75-cm-thick metamaterial sample, the branch indices over the: (a) 2–40 GHz range and (b) zoomed-in view on the 9–11-GHz frequency range.

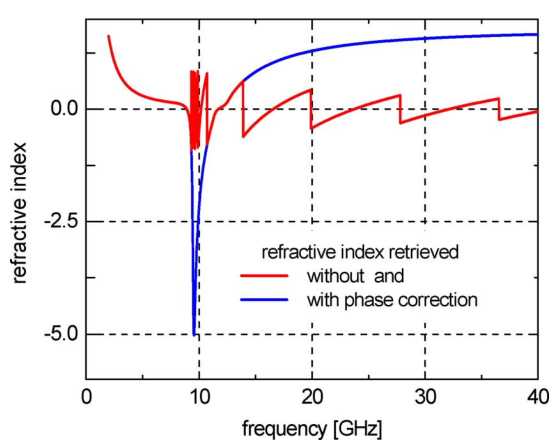

Fig. 3. Refractive index unambiguously retrieved (blue in online version); the index calculated without phase correction (red in online version).

been unambiguously determined, then we use (1) to calculate the corresponding refractive index (Fig. 3), thus without any discontinuities over the frequency range considered.

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Comments on "ParAFEMCap: A Parallel Adaptive Finite-Element Method for 3-D VLSI Interconnect Capacitance Extraction"

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The above paper [1] includes misleading statements on the accuracy and the efficiency of our technique [i.e., the characteristic basis finite-element method (CBFEM)], which was published in 2009 [2] for the extraction of 3-D capacitance matrices. The above paper [1] has devoted a separate section (i.e., Section II-B) to show the alleged weaknesses of our technique through some 2-D simulations. However, it seems that our technique was implemented erroneously in [1]. First of all, we would like to give brief information about our CBFEM technique, and then discuss why the claims in [1] are completely wrong.

The CBFEM is a relatively novel approach introduced to alleviate the difficulties of the conventional finite-element method (FEM) while solving large-scale electromagnetic boundary value problems. This is a matrix-reduction algorithm in the sense that it utilizes the strategy of domain decomposition by transforming the original matrix into a smaller one, referred to as the reduced matrix. For this purpose, characteristic basis functions (CBFs) are employed, which are high-level basis functions that are tailored in accordance with the physics of the problem under consideration. The first application of the CBFEM was to quasi-static problems, where it was used for the purpose of computing the capacitance matrices of 3-D interconnect structures by employing point charges to generate the CBFs [2]. Next, it was extended to the solution of electromagnetic scattering problems by using dipole-type sources with different approaches [3]–[6].

In the implementation of the CBFEM in [2], fictitious point charges are placed on the conductors, as shown in [2, Fig. 2]. The potentials created by these charges form the *natural* basis functions (i.e., CBFs) for the potential distribution within the entire computational domain. In other words, if the charge density is known along the boundaries of the conductors, then the potential distribution can be expressed as a convolution of the charge density along the conductors with the free-space Green's function that can be simply expressed as $1/4\pi\varepsilon R$. Afterwards, the CBFs are orthogonalized by using the singular value decomposition (SVD) approach. A threshold is then

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